Input Estimation From Measured Structural Response

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Abstract

This project will focus on the estimation of unmeasured inputs to a structure, given a numerical model of the structure and the structure's response. While the estimation of inputs has not received as much attention historically as state estimation, there are many applications where an improved understanding of the unmeasurable input to a structure is vital (e.g., validating load models for large structures such as buildings and ships). To accomplish this goal of input estimation, project participants will conduct a literature review on input estimation, implement and compare at least two estimation techniques (extended Kalman Filter and adjoint-based optimization), and validate their results on two mechanical test structures.

Project Description and Approach

The focus of the project is on the estimation of unmeasured inputs to a structure, given a numerical model of the structure and the structure's response. One approach to solving this problem would be guess an input, run a numerical simulation of the structuring using this input, and obtain an output, which could then be compared with the measured output. This process could be repeated until a satisfactory input is found. The advantage to this approach is its simplicity. The disadvantage is that it would likely take hundreds of thousands or even millions of simulations to find such an input, even for a simple structure. Fortunately there are better approaches.

The first approach is an extended Kalman filter [1], which allows one to estimate unmeasurable states in a dynamical system given limited observations. Without too much difficulty, the input could be treated as one of the states to be estimated. A potential disadvantage to this approach, which will have to be investigated by the project team, is that the confidence in the "early time" portion of the estimated input will be lower than the "late time" portion.

The second approach is known as adjoint-based optimization. With this technique, a simple cost function (based on the error between the predicted outputs and the measured outputs) is constructed, and the goal is to find an input which minimizes it. To accomplish this minimization, a fairly straight forward gradient-descent process will be followed. While this sounds like the above mentioned simplistic approach, the computational burden is avoided by efficiently being able to calculate the gradient. As is shown in very brief writeup in Appendix A, the gradient can be calculated with 2 simulations, regardless of how long the input is or how complicated the structure is (compare this to a finite difference approach for estimated the gradient which requires as many simulations as there are points in the input time history!).

While both approaches will be implemented in Matlab or Python and tested on simulated structures, it is important to conduct real-world validation. Thus, both techniques will be evaluated on two test structures. The first is nonlinear cantilever beam, shown in Figure

1, where the nonlinearity is introduced with magnets placed near the free end of the beam. The input is supplied via a shaker, and the response may be measured with accelerometers and/or a laser displacement sensor. The second test structure is a three story structure supported on roller bearings and excited by a shaker, as shown in Figure 2. Response is measured with accelerometers. For both sets of validation tests, the goal will be to see how well each of the two approaches reproduces the input signal.

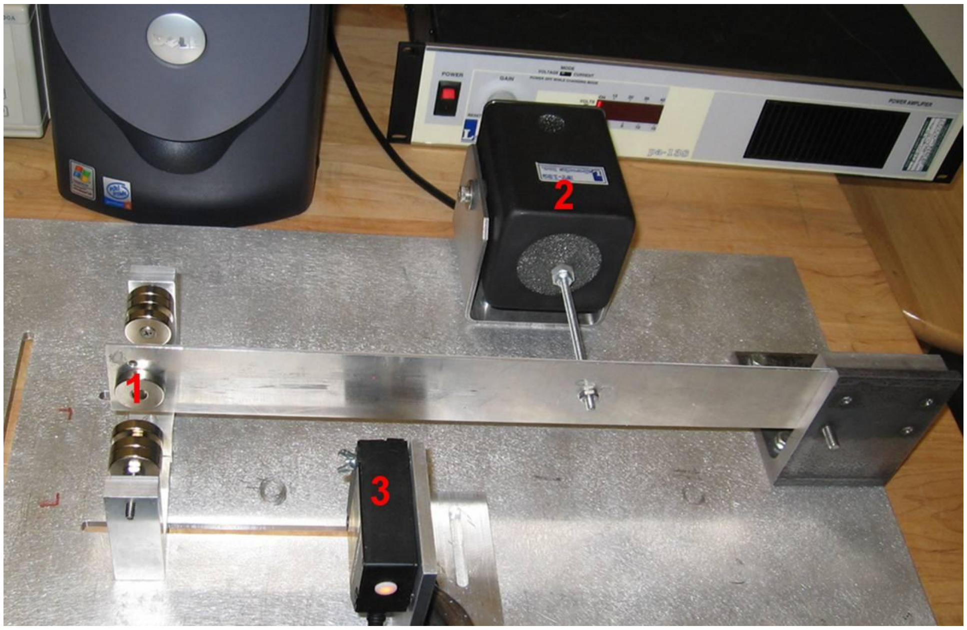

Figure 1. Cantilever beam test structure

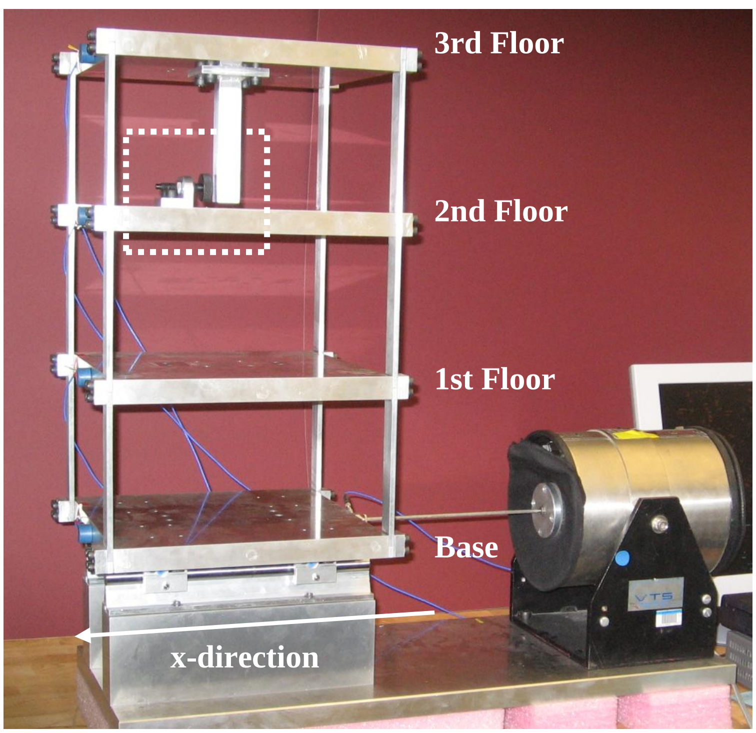

Figure 2. Three-story frame structure and shaker

Project Schedule

- Week 1: Complete safety training
- Week 2: Conduct literature review
- Week 3: Implement extended Kalman Filter for simulations
- Week 4: Implement adjoint-based optimization for simulations
- Week 5: Evaluate both techniques on cantilever beam
- Week 6: Evaluate both techniques on 3-story structure
- Week 7: Repeat any necessary experiments
- Week 8: Document results in the form of a conference paper
- Week 9: Present results

References

[1] http://en.wikipedia.org/wiki/Kalman_filter#Extended_Kalman_filter

Appendix A: Very Brief Summary of the Adjoint Method for Calculating a Gradient

Mathematically, the details of the adjoint-optimization approach are as follows. The nonlinear finite element model can be represented as

$$\dot{x} = f(x, u, t) \tag{1}$$

where x is the vector of system states (usually positions and velocities of all the nodes in the mesh of the structure), u is a vector of inputs at nodes on the structure and t is time. The measured outputs, y, can generally be represented as

$$y = Cx$$

where, C is some matrix. Stated another way, the outputs are some linear combination of the states of the system. The model's error is then defined to be

$$e = y - y_m$$

The goal of the optimization is to select u such that e is minimized. Or, more precisely, we want to minimize the cost function

$$J = \frac{1}{2} \int_0^T e^T e \, dt \tag{2}$$

Since $e = Cx - y_m$, and after some manipulation, this cost function can be rewritten as

$$J = \int_0^T x^T Q x - 2 y_m{}^T C x + y_m{}^T y_m \, dt$$

where $Q = C^T C$. If the input u is perturbed by u', the perturbed state trajectory is given by the tangent linear equation

$$\dot{x}' = A(t)x' + B(t)u' \quad \text{or} \quad \ell x' = Bu' \tag{3}$$

where $\ell = \frac{d}{dt} - A(t)$ and $A(t)$ and $B(t)$ are obtained by linearizing Equation 1 about x and u. The resulting perturbation to J is given by

$$J' = \int_0^T x^T Q x' - y_m{}^T C x' dt = \int_0^T \left(x^T Q - y_m{}^T C \right) x' dt \tag{4}$$

The goal of what follows is simply to re-express *J'* as a functional linear in *u'*. To that end, we integrate Equation (3) against a test function, *r*.

$$\int_0^T r^T \ell x' dt = \int_0^T r^T \left(x' - A(t)x'\right) dt$$

using integration by parts, we can rewrite the above equation as

$$\int_0^T r^T \ell x' dt = \int_0^T (\ell^* r)^T x' dt + r^T x' \Big|_{t=0}^{t=T}$$

where $\ell^* = -\frac{d}{dt} - A(t)^T$. This is true for any test function, *r*. If we select *r* such that

$$\begin{aligned} \ell^* r &= Qx - C^T y \\ r(T) &= 0 \end{aligned} \qquad (5)$$

then Equation 4 can be rewritten as

$$J' = \int_0^T \left(x^T Q - y_m{}^T C\right) x' dt = \int_0^T \left(\ell^* r\right)^T x' dt = \int_0^T r^T \ell x' dt = \int_0^T r^T B u' dt$$

Equation 5 is referred to as the adjoint equation. Thus, we have expressed *J'* as a functional linear in *u'*. The gradient of with respect to *u* is then simply

$$\frac{DJ}{Du} = r^T B$$

Therefore, givens some initial guess at *u*, Equation 1 is solved for *x*. This *x* is then used in conjunction with the measured data, y_m to solve Equation 5 in reverse-time, since *r(T)* is known. From *r*, the gradient of the cost function with respect to the input may be calculated, and used to update *u* (using any number of standard gradient descent based optimization techniques). Note that solving Equation 5 requires what is known as an adjoint version of the simulation code, which can calculate the linearized *A(t)* and *B(t)*, as functions of *x* and *u*.